Check-Up

19

Mental calculations, division strategies

Without resorting to a formal method like long division or a calculator, use a combination of mental calculations and informal jottings to find the number of pupils per teacher for each of these schools:

a) School A: 252 pupils, 12 teachers

b) School B: 805 pupils, 35 teachers

c) School C: 785 pupils, 44 teachers

Answers to check-up 19

a) 21. b) 23. c) nearly 18.

Possible methods are discussed below.

Discussion and explanation of check-up 19

We should be able to handle divisions of this level of difficulty without recourse to a calculator or formal written methods, such as long division. Informal methods are usually much easier than long division anyway. The calculations in this check-up illustrate some useful strategies.

For 252 ÷ 12, I might use the fact that you do not change the value of the ratio if you divide both numbers by the same thing. This is exactly the same as the process of 'cancelling' in fractions. In this example, I could easily halve each number to change the calculation to 126 ÷ 6. Halve them again: 63 ÷ 3. Then complete the calculation, thinking of the 63 as 60 + 3: 63 ÷ 3 = (60 ÷ 3) + (3 ÷ 3) = 20 + 1 = 21. This last step is using the distributive law (see Check-up 12).

You also do not change the value of a ratio if you multiply both numbers by the same thing. This is particularly useful when the number you are dividing by ends in a 5. For example, for 805 ÷ 35, I would double both numbers, to get 1610 ÷ 70. Then divide both by 10: 161 ÷ 7. Think of the 161 as 140 + 21: 161 ÷ 7 = (140 ÷ 7) + (21 ÷ 7) = 20 + 3.

Why did I think of changing 161 to 140 + 21? Because I am always on the lookout for numbers that divide easily. If I have to divide by 7 then the number closest to 161 that looks friendly is 140.

So, given 785 ÷ 44, I would think: I wish it was 880 ÷ 44, because that would just be 20. But that would be using 95 too many pupils. Now I think: I wish that 95 was 88, because 88 ÷ 44 = 2. But that would be using 7 too many pupils. So if there were just 7 more pupils, the ratio would be 20 – 2 = 18. The total of 7 pupils we are missing to give us the pupil–teacher ratio of 18 is less than half a pupil for each one of the 44 teachers, so clearly the PTR rounded to the nearest whole number is 18. If you need the result to be more precise than this then you should probably use a calculator.

In other contexts, the answer to 785 ÷ 44 might have been 17. For example, how many graphic calculators costing £44 each can you buy with £785? In this

case you are £7 short of what you need for 18 calculators, so you can actually only afford 17, leaving you with £35 left to spend.

Summary of key ideas

- It is usually easier to do a division by a one- or two-digit number using informal methods than using a formal process like long division.
- One way of simplifying a division calculation is to multiply or divide both numbers involved by the same thing (e.g. change 225 ÷ 15 to 450 ÷ 30, by doubling; change 450 ÷ 30 to 45 ÷ 3 by dividing by 10).
- Another strategy is to look for easier, related divisions, by relating the number you are dividing to a combination of numbers that are easier to divide, using addition and subtraction (e.g. for 198 ÷ 9 change the 198 to 180 + 18; for 198 ÷ 22 change the 198 to 220 – 22).

Further practice

The questions here illustrate the level of division calculations you should be able to manage without a calculator, using informal jottings and mental calculations. The answers provide just some suggestions for how to tackle these – you may have better methods.

19.1 The QCA report on the 1998 Key Stage 3 mathematics National Curriculum assessments comments that most pupils attempted to calculate 144 ÷ 9 by using a formal written division method. How else might they have done it more easily?

19.2 A small primary school with 85 pupils spends £6035 on learning resources one year. How much is this per pupil?

19.3 To find the average mark achieved by pupils in an English test, the teacher adds up all the marks (total 893) and divides by the number of pupils (24). What is the result to the nearest whole number?